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Quasibosons

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The similarity of the commutation relations for bosons and quasibosons (fermion pairs) suggests the possibility that all integral spin particles presently considered to be bosons could be quasibosons. The boson commutation relations for integral spin particles could be just an approximation to the quasiboson commutation relations that contain an extra term. Although the commutation relations for quasibosons are slightly more complex, it is a simpler picture of matter in that only fermions and composite particles formed of fermions exist. Mesons are usually referred to as bosons, but they must be quasibosons since their internal structure is fermion (quark) pairs. The photon is usually considered to be an elementary boson, but as shown here, existing experiments do not rule out the possibility that it is also a quasiboson. We consider how the quasiboson, composite nature of such a photon might manifest itself.

KEY WORDS: quasibosons; composite photon; commutation relations; symmetry under interchange.

1. INTRODUCTION

Most integral spin particles (light mesons, strange mesons, etc.) are composite particles formed of quarks. Because of their underlying fermion structure, these integral spin particles are not fundamental bosons, but composite quasibosons. However, in the asymptotic limit, which generally applies, they are essentially bosons. For these particles, Bose commutation relations are just an approximation, albeit a very good one. There are some differences; bringing two of these composite particles close together will force their identical fermions to jump to excited states because of the Pauli exclusion principle.

A few integral spin particles (photons, gluons, weak bosons, and gravitons) are regarded as elementary, exact bosons. For all of these particles, except the photon, there is no direct evidence from their statistics to differentiate between boson and quasiboson behavior. As we shall show in this paper, surprisingly, present experiments do not differentiate between a "boson photon" and a "quasiboson photon." If the photon were a quasiboson, it would presumable be composed of neutrinos or massless quarks.

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Since the predictions of quantum electrodynamics are in such excellent agreement with experiment (e.g., the calculated anomalous magnetic moment of the electron agrees to a few parts per billion with experiment), one might think there is little or no room for improvement in the model of the photon. In addition, since the photon is known to be massless to high precision, how can it be a composite particle? Would not a composite photon have readily detectable self-interactions? Does not the association of the photon with local gauge invariance indicate that it is elementary?

In spite of all its successes, there are some problems with the present photon model: Many of the calculations diverge (e.g., the calculated anomalous magnetic moment of the electron really gives infinity, before renormalization) and nonphysical photon polarization states must be introduced to satisfy Lorentz invariance. Some of these problems are discussed in a recent paper concerning composite photons (Perkins, 2000). A method of binding a fermion–antifermion pair with a zero range interaction that does not involve quanta is also discussed (Perkins, 2000). With such an interaction the composite particle need not have mass or self-interactions. It is further shown (Perkins, 2000) that a composite-photon theory can be Lorentz invariant without the need for gauge invariance.

Let us compare the commutation relations for fermions, bosons, and quasibosons. Fermions are defined as the particles whose creation and annihilation operators obey the anticommutation relations,

$$\begin{aligned} \{a(\mathbf{k}), a(\mathbf{l})\} &= 0, \\ \{a^\dagger(\mathbf{k}), a^\dagger(\mathbf{l})\} &= 0, \\ \{a(\mathbf{k}), a^\dagger(\mathbf{l})\} &= \delta(\mathbf{k}-\mathbf{l}), \end{aligned} \tag{1}$$

while bosons are defined as the particles that obey the commutation relations,

$$\begin{aligned} [b(\mathbf{k}), b(\mathbf{l})] &= 0, \\ [b^\dagger(\mathbf{k}), b^\dagger(\mathbf{l})] &= 0, \\ [b(\mathbf{k}), b^\dagger(\mathbf{l})] &= \delta(\mathbf{k}-\mathbf{l}). \end{aligned} \tag{2}$$

In superconductivity (Blatt, 1964), the Cooper pairs are referred to as "quasibosons," since they obey commutation relations similar to, but different from, those of bosons. It is well known that molecules with an even number of fermions exhibit Bose behavior, while those with an odd number exhibit Fermi behavior (Ehrenfest and Oppenheimer, 1931). Theoretically, these composite molecules formed of an even number of fermions (as well as nuclei formed of an even number of fermions) do not obey Bose commutation relations and so we will refer to them as quasibosons.

The creation and annihilation operators of quasibosons (composite particles formed of fermion pairs) obey the commutation relations of the form

$$\begin{aligned} [Q(\mathbf{k}), Q(\mathbf{l})] &= 0, \\ [Q^{\dagger}(\mathbf{k}), Q^{\dagger}(\mathbf{l})] &= 0, \\ [Q(\mathbf{k}), Q^{\dagger}(\mathbf{l})] &= \delta(\mathbf{k}-\mathbf{l}) - \Delta(\mathbf{k}, \mathbf{l}). \end{aligned} \tag{3}$$

The commutation relations for a pair of fermions, Eq. (3), are similar to those for bosons, Eq. (2). The $\Delta(\mathbf{k}, \mathbf{l})$ term (see Eq. (9)) looks complicated but its value is usually very small. Thus, it is easy to envisage that Eq. (2) is just an approximation to Eq. (3), the more accurate commutation relations for integral spin particles.

As presented in many quantum mechanics texts it may appear that Bose statistics follow from basic principles, but it is really from the classical canonical formalism. This is not a reliable procedure as evidenced by the fact that it gives the completely wrong result for spin-$\frac{1}{2}$ particles. Furthermore, in extending the classical canonical formalism for the photon, it is necessary to deviate from the canonical rules (see Bjorken and Drell, 1965, pp. 71, 98).

Based on the symmetry of systems of identical particles, it can be shown that their wavefunctions must be symmetric or antisymmetric under interchange (see Bjorken and Drell, 1965, pp. 32–34). Although identical bosons are symmetric under interchange, so are identical quasibosons. It has also been claimed that the spin-statistics theorem requires that integral spin particles must be bosons. In Section 5, the spin-statistics theorem is reviewed and shown not to apply to quasibosons which do not satisfy space-like commutativity because of their finite extent.

In Section 6, the experimental evidence concerning the photon is examined. We conclude that present experiments do not rule out the possibility that photons are quasibosons. Although the state of two elementary neutral bosons (which are their own antiparticle and are identical) must be symmetric under exchange, two composite quasibosons (which are not identical) can be antisymmetric or symmetric under interchange. The author has suggested (Perkins, 1999) that there exist two distinct π^0s as a solution to the $\bar{p}p \to \pi^0\pi^0$ puzzle. As discussed in Section 3, composite photons can be nonidentical also, making possible axial vector meson decays into two photons (see Section 6.2).

2. STATISTICS OF QUASIBOSONS

Consider quasibosons formed of two different types of fermions of equal mass whose annihilation operators are given the symbols "a" and "c." Quasibosons, formed from pairs of spin-$\frac{1}{2}$ particles, have spin 0 or 1. Assuming that the system is in a large box of finite volume with periodic boundary conditions, the quasiboson annihilation and creation operators are defined as (Landau, 1996; Lipkin, 1973;

Perkins, 1972; Sahlin and Schwartz, 1965)

$$\begin{aligned} Q(\mathbf{p}) &= \sum_{\mathbf{k}} F^\dagger(\mathbf{k}) c(\mathbf{p}/2 - \mathbf{k}) a(\mathbf{p}/2 + \mathbf{k}), \\ Q^\dagger(\mathbf{p}) &= \sum_{\mathbf{k}} F(\mathbf{k}) a^\dagger(\mathbf{p}/2 + \mathbf{k}) c^\dagger(\mathbf{p}/2 - \mathbf{k}), \end{aligned} \tag{4}$$

where $a(\mathbf{k})$ and $c(\mathbf{k})$ are annihilation operators for two different types of fermions.

The Fourier transform of the creation operator is

$$Q^\dagger(\mathbf{R}) = \int d\mathbf{r}\, \phi(\mathbf{r}) \Psi_a^\dagger(\mathbf{R} - \mathbf{r}/2) \Psi_c^\dagger(\mathbf{R} + \mathbf{r}/2), \tag{5}$$

where $\phi(\mathbf{r})$ describes the relative motion and can be expanded in plane waves,

$$\phi(\mathbf{r}) = \sum_{\mathbf{k}} F(\mathbf{k})\, e^{-i\mathbf{kr}}. \tag{6}$$

The relative and center-of-mass coordinates and momenta are defined by

$$\begin{aligned} \mathbf{r} &= \mathbf{r_1} - \mathbf{r_2}, \qquad \mathbf{R} = \frac{\mathbf{r_1} + \mathbf{r_2}}{2}, \\ \mathbf{k} &= \frac{\mathbf{k_1} - \mathbf{k_2}}{2}, \qquad \mathbf{p} = \mathbf{k_1} + \mathbf{k_2}, \end{aligned} \tag{7}$$

with

$$\sum_{\mathbf{k}} |F(\mathbf{k})|^2 = 1. \tag{8}$$

The quasiboson operators of Eq. (4) have been shown (Landau, 1996; Lipkin, 1973; Perkins, 1972; Sahlin and Schwartz, 1965) to obey the commutation relations (3) with

$$\begin{aligned} \Delta(\mathbf{p}', \mathbf{p}) = \sum_{\mathbf{k}} F^\dagger(\mathbf{k}) [&F(\mathbf{p}'/2 - \mathbf{p}/2 + \mathbf{k}) a^\dagger(\mathbf{p} - \mathbf{p}'/2 - \mathbf{k}) a(\mathbf{p}'/2 - \mathbf{k}) \\ &+ F(\mathbf{p}/2 - \mathbf{p}'/2 + \mathbf{k}) c^\dagger(\mathbf{p} - \mathbf{p}'/2 + \mathbf{k}) c(\mathbf{p}'/2 + \mathbf{k})]. \end{aligned} \tag{9}$$

For Cooper electron pairs (Landau, 1996), a and c represent different spin directions. For nucleon pairs (the deuteron) (Landau, 1996; Lipkin, 1973), a and c represent proton and neutron. For neutrino–antineutrino pairs (Perkins, 1972), a and c represent neutrino and antineutrino. The size of the deviations from pure Bose behavior, $\Delta(\mathbf{p}', \mathbf{p})$, depends on the degree of overlap of the fermion wave functions and the constraints of the Pauli principle.

If we assume the state has the form

$$|\Phi\rangle = a^\dagger(\mathbf{k_1}) a^\dagger(\mathbf{k_2}) \cdots a^\dagger(\mathbf{k_n}) c^\dagger(\mathbf{q_1}) c^\dagger(\mathbf{q_2}) \cdots c^\dagger(\mathbf{q_m}) |0\rangle \tag{10}$$

then the expectation value of (9) vanishes for $\mathbf{p}' \neq \mathbf{p}$, and we can approximate the expression for $\Delta(\mathbf{p}', \mathbf{p})$ by

$$\Delta(\mathbf{p}', \mathbf{p}) = \delta(\mathbf{p}' - \mathbf{p}) \sum_{\mathbf{k}} |F(\mathbf{k})|^2 [a^\dagger(\mathbf{p}/2 - \mathbf{k}) a(\mathbf{p}/2 - \mathbf{k}) + c^\dagger(\mathbf{p}/2 + \mathbf{k}) c(\mathbf{p}/2 + \mathbf{k})]. \tag{11}$$

Using the fermion number operators $n_a(\mathbf{k})$ and $n_c(\mathbf{k})$, this can be written as

$$\begin{aligned} \Delta(\mathbf{p}', \mathbf{p}) &= \delta(\mathbf{p}' - \mathbf{p}) \sum_{\mathbf{k}} |F(\mathbf{k})|^2 [n_a(\mathbf{p}/2 - \mathbf{k}) + n_c(\mathbf{p}/2 + \mathbf{k})] \\ &= \delta(\mathbf{p}' - \mathbf{p}) \sum_{\mathbf{k}} [|F(\mathbf{p}/2 - \mathbf{k})|^2 n_a(\mathbf{k}) + |F(\mathbf{k} - \mathbf{p}/2)|^2 n_c(\mathbf{k})] \\ &= \delta(\mathbf{p}' - \mathbf{p}) \bar{\Delta}(\mathbf{p}, \mathbf{p}) \end{aligned} \tag{12}$$

showing that it is the average number of fermions in a particular state $\mathbf{k}$ averaged over all states with weighting factors $F(\mathbf{p}/2 - \mathbf{k})$ and $F(\mathbf{k} - \mathbf{p}/2)$.

The number operator for quasibosons is defined as

$$N(\mathbf{p}) = Q^\dagger(\mathbf{p}) Q(\mathbf{p}). \tag{13}$$

Using (3), (9), and (13), we obtain the following commutation relations for the number operator:

$$\begin{aligned} [N(\mathbf{p}'), Q(\mathbf{p})] &= -\{\delta(\mathbf{p}' - \mathbf{p}) - \Delta(\mathbf{p}', \mathbf{p})\} Q(\mathbf{p}'), \\ [N(\mathbf{p}'), Q^\dagger(\mathbf{p})] &= Q^\dagger(\mathbf{p}')\{\delta(\mathbf{p}' - \mathbf{p}) - \Delta(\mathbf{p}', \mathbf{p})\}, \\ [N(\mathbf{p}'), N(\mathbf{p})] &= Q^\dagger(\mathbf{p}) \Delta(\mathbf{p}', \mathbf{p}) Q(\mathbf{p}') - Q^\dagger(\mathbf{p}') \Delta(\mathbf{p}', \mathbf{p}) Q(\mathbf{p}), \\ \langle [N(\mathbf{p}'), N(\mathbf{p})] \rangle &= 0. \end{aligned} \tag{14}$$

As expected, these commutation relations differ from the usual Bose relations by terms involving $\Delta(\mathbf{p}', \mathbf{p})$. Note that $\Delta(\mathbf{p}', \mathbf{p})$ does not commute with the quasiboson annihilation and creation operators. Using the second equation of (12), we obtain

$$[\bar{\Delta}(\mathbf{p}, \mathbf{p}), Q^\dagger(\mathbf{q})] = \sum_{\mathbf{k}} \{|F(\mathbf{p}/2 - \mathbf{k})|^2 [n_a(\mathbf{k}), Q^\dagger(\mathbf{q})] + |F(\mathbf{k} - \mathbf{p}/2)|^2 [n_c(\mathbf{k}), Q^\dagger(\mathbf{q})]\}. \tag{15}$$

Inserting (4) for $Q^\dagger(\mathbf{q})$ and using the usual commutation relations for the fermion number operator (see Landau, 1996, p. 456) gives

$$[\bar{\Delta}(\mathbf{p}, \mathbf{p}), Q^\dagger(\mathbf{q})] = \sum_{\mathbf{k}} \{|F(\mathbf{p}/2 - \mathbf{q}/2 - \mathbf{k})|^2 + |F(\mathbf{q}/2 - \mathbf{p}/2 - \mathbf{k})|^2\} \times F(\mathbf{k}) a^\dagger(\mathbf{q}/2 + \mathbf{k}) c^\dagger(\mathbf{q}/2 - \mathbf{k}). \tag{16}$$

At this point we need an approximation to obtain a workable value. Lipkin (1973) suggested for a rough estimate to assume that $F(\mathbf{k})$ is a constant for the states used to construct the wave packet. He used the symbol Ω for the number of states used to construct the wave packet. In that case, Eq. (8) gives $|F(\mathbf{k})|^2 = 1/\Omega$, and one obtains directly from (16),

$$[\bar{\Delta}(\mathbf{p}, \mathbf{p}), Q^\dagger(\mathbf{q})] = 2Q^\dagger(\mathbf{q})/\Omega(\mathbf{p}, \mathbf{q}). \tag{17}$$

In Lipkin's approximation, Ω does not depend upon $\mathbf{p}$ and $\mathbf{q}$. An improvement can be made by using

$$\frac{1}{\Omega(\mathbf{p}, \mathbf{q})} = \frac{\sum_{\mathbf{k}} F(\mathbf{k})\{|F(\mathbf{p}/2 - \mathbf{q}/2 - \mathbf{k})|^2 + |F(\mathbf{q}/2 - \mathbf{p}/2 - \mathbf{k})|^2\}}{2\sum_{\mathbf{k}} F(\mathbf{k})}, \tag{18}$$

and letting $F(\mathbf{k})$ be a Gaussian distribution, which satisfies the normalization condition (8)

$$F(\mathbf{k}) = \frac{(8\pi)^{3/4}}{\sqrt{Vk_0^3}} e^{-k^2/k_0^2}, \tag{19}$$

where V is the confinement volume. Going from a box to the infinite domain, Eq. (18) becomes

$$\frac{1}{\Omega(\mathbf{p}, \mathbf{q})} = \frac{\int d^3k\, F(\mathbf{k})|F(\mathbf{p}/2 - \mathbf{q}/2 - \mathbf{k})|^2}{\int d^3k\, F(\mathbf{k})}. \tag{20}$$

Inserting the Gaussian distribution for $F(\mathbf{k})$ and evaluating the integrals results in

$$\Omega(\mathbf{p}, \mathbf{q}) = \left(\frac{3}{8\pi}\right)^{3/2} V \left(\frac{k_0}{\hbar c}\right)^3 e^{(\mathbf{p}-\mathbf{q})^2/(6k_0^2)}. \tag{21}$$

We can see from (18) that $1/\Omega(\mathbf{p}, \mathbf{q})$ will be very small if $F(\mathbf{k})$ and $F(\mathbf{p}/2 - \mathbf{q}/2 - \mathbf{k})$ have little overlap. This can occur if $\mathbf{q} = -\mathbf{p}$ (two quasibosons emitted in opposite directions) and $|\mathbf{p}| \gg k_0$, the Gaussian width or momentum spread. This overlap factor is given explicitly in the exponential of (21). For the case in which $\mathbf{q} = \mathbf{p}$, $\Omega(\mathbf{p}, \mathbf{q})$ does not depend upon $\mathbf{p}$, so we can just use Ω.

We can now use the second equation of (14) and (17) to find the effect of the quasiboson number operator acting on a state of m quasibosons

$$N(\mathbf{p})(Q^\dagger(\mathbf{p}))^m|0\rangle = \left(m - \frac{m(m-1)}{\Omega(\mathbf{p}, \mathbf{p})}\right)(Q^\dagger(\mathbf{p}))^m|0\rangle, \tag{22}$$

where we have used $N(\mathbf{p})|0\rangle = \bar{\Delta}(\mathbf{p}, \mathbf{p})|0\rangle = 0$. This result differs from the usual one because of the second term which is small for large Ω. Normalizing in the usual manner (see Koltun and Eisenberg, 1988, p. 7),

$$\begin{aligned} Q^{\dagger}(\mathbf{p})|n_{\mathbf{p}}\rangle &= \sqrt{(n_{\mathbf{p}}+1)\left(1-\frac{n_{\mathbf{p}}}{\Omega}\right)}|n_{\mathbf{p}}+1\rangle, \\ Q(\mathbf{p})|n_{\mathbf{p}}\rangle &= \sqrt{n_{\mathbf{p}}\left(1-\frac{(n_{\mathbf{p}}-1)}{\Omega}\right)}|n_{\mathbf{p}}-1\rangle, \end{aligned} \tag{23}$$

where $|n_{\mathbf{p}}\rangle$ is the state of $n_{\mathbf{p}}$ quasibosons having momentum $\mathbf{p}$ which is created by applying $Q^{\dagger}(\mathbf{p})$ on the vacuum $n_{\mathbf{p}}$ times. Note that

$$\begin{aligned} Q^{\dagger}(\mathbf{p})|0\rangle &= |1_{\mathbf{p}}\rangle, \\ Q(\mathbf{p})|1_{\mathbf{p}}\rangle &= |0\rangle, \end{aligned} \tag{24}$$

which is the same result as obtained with boson (or fermion) operators. In Eq. (23) we see formulas similar to the usual ones with correction factors that approach zero for large Ω.

Now, let us look at possible commutation relations for a spin-1 quasiboson photon. The quasiboson annihilation and creation operators need to be modified slightly to handle spin and a mass zero composite particle. We consider photons to be composed of two-component neutrinos and their antiparticle (momenta antiparallel and spins parallel). The neutrino and antineutrino are assumed to have antiparallel momentum when created and absorbed in interactions with other particles. The photon is continuously creating virtual pairs that annihilate. (Whether this is the correct composition for a real photon is not of concern here. Our main interest is to have a model of a quasiboson photon for comparison purposes.) Let $\gamma_{\mathrm{R}}(\mathbf{p})$ and $\gamma_{\mathrm{L}}(\mathbf{p})$ be annihilation operators for right and left circularly polarized photons. Then,

$$\begin{aligned} \gamma_{\mathrm{R}}(\mathbf{p}) &= \frac{1}{\sqrt{2}}\sum_{\mathbf{k}} F^{\dagger}(k,\mathbf{n})[c_1(k,-\mathbf{n})a_1(p+k,\mathbf{n}) + c_2(p+k,\mathbf{n})a_2(k,-\mathbf{n})], \\ \gamma_{\mathrm{L}}(\mathbf{p}) &= \frac{1}{\sqrt{2}}\sum_{\mathbf{k}} F^{\dagger}(k,\mathbf{n})[c_2(k,-\mathbf{n})a_2(p+k,\mathbf{n}) + c_1(p+k,\mathbf{n})a_1(k,-\mathbf{n})], \end{aligned} \tag{25}$$

where $\mathbf{n} = \mathbf{p}/|\mathbf{p}| = \mathbf{k}/|\mathbf{k}|$ and a corresponds to the neutrino and c, the antineutrino while the subscripts refer to different two-component neutrinos. Another complexity, multiple two-fermion states, has also been introduced in Eq. (25). With the same approximation as used in Eq. (11), the commutation relations for $\gamma_{\mathrm{R}}(\mathbf{p})$ and $\gamma_{\mathrm{L}}(\mathbf{p})$ become

$$\begin{aligned} [\gamma_{\mathrm{R}}(\mathbf{p}'), \gamma_{\mathrm{R}}(\mathbf{p})] &= 0, \\ [\gamma_{\mathrm{L}}(\mathbf{p}'), \gamma_{\mathrm{L}}(\mathbf{p})] &= 0, \end{aligned}$$

$$
\begin{aligned}
[\gamma_{\mathrm{R}}(\mathbf{p}'), \gamma_{\mathrm{R}}^{\dagger}(\mathbf{p})] &= \delta(\mathbf{p}' - \mathbf{p})(1 - \bar{\Delta}_{12}(\mathbf{p}, \mathbf{p})), \\
[\gamma_{\mathrm{L}}(\mathbf{p}'), \gamma_{\mathrm{L}}^{\dagger}(\mathbf{p})] &= \delta(\mathbf{p}' - \mathbf{p})(1 - \bar{\Delta}_{21}(\mathbf{p}, \mathbf{p})), \\
[\gamma_{\mathrm{R}}(\mathbf{p}'), \gamma_{\mathrm{L}}(\mathbf{p})] &= 0, \\
[\gamma_{\mathrm{R}}(\mathbf{p}'), \gamma_{\mathrm{L}}^{\dagger}(\mathbf{p})] &= 0,
\end{aligned}
\tag{26}
$$

where

$$
\begin{aligned}
\bar{\Delta}_{12}(\mathbf{p}, \mathbf{p}) = \frac{1}{2} \sum_{\mathbf{k}} |F(k, \mathbf{n})|^2 [&a_1^{\dagger}(p + k, \mathbf{n}) a_1(p + k, \mathbf{n}) + c_1^{\dagger}(k, -\mathbf{n}) c_1(k, -\mathbf{n}) \\
&+ c_2^{\dagger}(p + k, \mathbf{n}) c_2(p + k, \mathbf{n}) + a_2^{\dagger}(k, -\mathbf{n}) a_2(k, -\mathbf{n})].
\end{aligned}
\tag{27}
$$

We now consider whether dissimilar quasibosons commute or anticommute. In general, two unlike quasibosons will commute as pairs of nonidentical fermions commute. However, if two dissimilar quasibosons contain the same fermion(s), then they will commute with commutation relations similar to Eq. (3) with the $\Delta(\mathbf{k}, \mathbf{l})$ term modified by some appropriate factor.

3. SYMMETRY OF QUASIBOSONS UNDER INTERCHANGE

First we consider a state of two identical quasibosons. Assuming that the system is in a large box of finite volume with periodic boundary conditions, a state of two quasibosons in the formalism of Eq. (4) is represented by

$$
|Q_1 Q_2\rangle = \sum_{\mathbf{p_1}, \mathbf{p_2}} f(\mathbf{p_1}, \mathbf{p_2}) Q^{\dagger}(\mathbf{p_1}) Q^{\dagger}(\mathbf{p_2}) |0\rangle. \tag{28}
$$

Since the quasiboson creation operators commute, the state of two quasibosons as given by Eq. (28) is symmetric under interchange of the quasibosons.

While the state of two identical bosons must be symmetric under interchange, a state of nonidentical bosons (or quasibosons) can be either symmetric or antisymmetric. For example, a state of two identical π^0s must be a state with even relative orbital angular momentum, while a state of $\pi^+\pi^-$ can have even or odd relative orbital angular momentum.

We will consider the symmetry of a state of two quasiboson photons. Linearly polarized photon annihilation operators can be constructed from circularly polarized operators by

$$
\begin{aligned}
\xi(\mathbf{p}) &= \frac{1}{\sqrt{2}} [\gamma_{\mathrm{L}}(\mathbf{p}) + \gamma_{\mathrm{R}}(\mathbf{p})], \\
\eta(\mathbf{p}) &= \frac{i}{\sqrt{2}} [\gamma_{\mathrm{L}}(\mathbf{p}) - \gamma_{\mathrm{R}}(\mathbf{p})].
\end{aligned}
\tag{29}
$$

Using Eq. (25) we obtain

$$
\begin{aligned}
\xi(\mathbf{p}) = \frac{1}{2} \sum_{\mathbf{k}} F^{\dagger}(k, \mathbf{n})[&c_1(k, -\mathbf{n}) a_1(p+k, \mathbf{n}) + c_2(p+k, \mathbf{n}) a_2(k, -\mathbf{n}) \\
&+ c_2(k, -\mathbf{n}) a_2(p+k, \mathbf{n}) + c_1(p+k, \mathbf{n}) a_1(k, -\mathbf{n})], \\
\eta(\mathbf{p}) = \frac{i}{2} \sum_{\mathbf{k}} F^{\dagger}(k, \mathbf{n})[&c_1(k, -\mathbf{n}) a_1(p+k, \mathbf{n}) + c_2(p+k, \mathbf{n}) a_2(k, -\mathbf{n}) \\
&- c_2(k, -\mathbf{n}) a_2(p+k, \mathbf{n}) - c_1(p+k, \mathbf{n}) a_1(k, -\mathbf{n})].
\end{aligned} \tag{30}
$$

From Eq. (30) we see that a state of two composite photons such as

$$
|\Phi\rangle = \sum_{\mathbf{p_1}, \mathbf{p_2}} f(\mathbf{p_1}, \mathbf{p_2}) \xi^{\dagger}(\mathbf{p_1}) \eta^{\dagger}(\mathbf{p_2}) |0\rangle \tag{31}
$$

need not be symmetric under interchange as the two photons are not identical.

4. COMMUTATION RELATIONS FOR QUASIBOSON FIELDS

Consider the field

$$
\phi(x) = A(x) + A^{\dagger}(x), \tag{32}
$$

where

$$
A(x) = \frac{1}{(2\pi)^3} \int d^3p \, \frac{1}{\sqrt{2p_0}} Q(\mathbf{p}) \, e^{ipx}, \tag{33}
$$

with $px = \mathbf{p} \cdot \mathbf{x} - p_0 x_0$. We will follow the usual argument, which shows that Bose fields are local (e.g., see Veltman, 1994, p. 25), except that Eq. (33) has a quasiboson operator $Q(\mathbf{p})$ instead of a Bose operator. Using Eqs. (3) and (12) for the quasiboson commutation relations gives

$$
[A(x), A^{\dagger}(y)] = \frac{1}{(2\pi)^3} \int d^3p \, \frac{1}{\sqrt{2p_0}} e^{ip(x-y)} [1 - \bar{\Delta}(\mathbf{p}, \mathbf{p})]. \tag{34}
$$

Since

$$
[A^{\dagger}(x), A(y)] = -[A(y), A^{\dagger}(x)], \tag{35}
$$

Eqs. (32) and (34) result in

$$
[\phi(x), \phi(y)] = \frac{1}{(2\pi)^3} \int d^3p \, \frac{1}{2p_0} \left\{ e^{ip(x-y)} - e^{-ip(x-y)} \right\} [1 - \bar{\Delta}(\mathbf{p}, \mathbf{p})], \tag{36}
$$

which can be expressed as

$$
[\phi(x), \phi(y)] = iD(x-y) - \frac{i}{(2\pi)^3} \int d^3p \, \frac{1}{p_0} \, \sin[p(x-y)] \bar{\Delta}(\mathbf{p}, \mathbf{p}). \tag{37}
$$

For equal times $D(\mathbf{x}-\mathbf{y}, 0)$ is zero, but because of the second term this commutator does *not* vanish for space-like intervals $(x-y)^2 < 0$. This means that quasibosons, being composite particles, have a finite extent in space. Quasiboson fields cannot have a commutator which vanishes everywhere outside the light cone. Otherwise, one could prove using the spin-statistics theorem that quasibosons are bosons, a contradiction.

This field satisfies the Klein–Gordon equation and therefore might be appropriate for a spin-0 composite particle. The nonlocal field effect carries over to quasibosons with spin, of course.

The commutation relations for the electromagnetic fields of quasibosons in terms of $\bar{\Delta}_{12}(\mathbf{p}, \mathbf{p})$ and $\bar{\Delta}_{21}(\mathbf{p}, \mathbf{p})$ of Eq. (27) are (see Appendix A of Perkins, 1965)

$$\begin{aligned}
[E_i(x), E_j(y)] = {} & \left(\delta_{ij}\frac{\partial}{\partial x_0}\frac{\partial}{\partial y_0} - \frac{\partial}{\partial x_i}\frac{\partial}{\partial y_j}\right)\left\{iD(x-y) - \frac{i}{16\pi^3}\int d^3p\, p_0^{-1}\right.\\
& \left.\times \sin[p(x-y)](\bar{\Delta}_{12}(\mathbf{p},\mathbf{p}) + \bar{\Delta}_{21}(\mathbf{p},\mathbf{p}))\right\} - \frac{i}{16\pi^3}\frac{\partial}{\partial y_0}\sum_{k=1}^{3}\epsilon_{ijk}\\
& \times \frac{\partial}{\partial x_k}\int d^3p\, p_0^{-1}\cos[p(x-y)](\bar{\Delta}_{12}(\mathbf{p},\mathbf{p}) - \bar{\Delta}_{21}(\mathbf{p},\mathbf{p})),
\end{aligned} \tag{38}$$

$$[H_i(x), H_j(y)] = [E_i(x), E_j(y)], \tag{39}$$

and

$$\begin{aligned}
[E_i(x), H_j(y)] = {} & -\frac{\partial}{\partial y_0}\sum_{k=1}^{3}\epsilon_{ijk}\frac{\partial}{\partial x_k}\left\{iD(x-y) - \frac{i}{16\pi^3}\int d^3p\, p_0^{-1}\right.\\
& \left.\times \sin[p(x-y)](\bar{\Delta}_{12}(\mathbf{p},\mathbf{p}) + \bar{\Delta}_{21}(\mathbf{p},\mathbf{p}))\right\} - \frac{i}{16\pi^3}\\
& \times \left(\delta_{ij}\frac{\partial}{\partial x_0}\frac{\partial}{\partial y_0} - \frac{\partial}{\partial x_i}\frac{\partial}{\partial y_j}\right)\int d^3p\, p_0^{-1}\cos[p(x-y)]\\
& \times (\bar{\Delta}_{12}(\mathbf{p},\mathbf{p}) - \bar{\Delta}_{21}(\mathbf{p},\mathbf{p})).
\end{aligned} \tag{40}$$

These quasiboson commutation relations differ from the usual photon commutation relations because of the extra terms involving $\bar{\Delta}_{12}(\mathbf{p}, \mathbf{p})$ and $\bar{\Delta}_{21}(\mathbf{p}, \mathbf{p})$. To estimate the deviation from local commutativity, we note that $\bar{\Delta}(\mathbf{p}, \mathbf{p})$ is independent of $\mathbf{p}$ in Lipkin's approximation (see Eq. (6.13) of Lipkin, 1973), and so it can be taken out from under the integral sign. With that change, the "sin" terms in Eqs. (37), (38), and (40) contain the factor $D(x-y)$ which vanishes for space-like intervals. The "cos" terms fall off as $1/|\mathbf{x}-\mathbf{y}|^2$ for large $\mathbf{x}-\mathbf{y}$ for a mass-zero particle (see Bjorken and Drell, 1965, p. 171), and the factor $\bar{\Delta}_{12}(\mathbf{p}, \mathbf{p}) - \bar{\Delta}_{21}(\mathbf{p}, \mathbf{p})$

should be small as it vanishes for equal numbers of right-handed and left-handed photons.

The departure from local commutativity allows an interference between a particle created at **x** and one created at **y**, but does not restrict the measurability of $\phi(\mathbf{x})$ or $\phi^{\dagger}(\mathbf{x})$. Similarly, the small interference indicated by Eqs. (38), (39), and (40) should not significantly affect the measurability of the fields as long as we do not attach physical meaning to the measurement of the field strength at a point, but to averages over finite space-time regions (Bohr and Rosenfeld, 1950).

5. SPIN-STATISTICS THEOREM

In his 1940 paper, Pauli (1940) concludes: "For integral spin the quantization according to the exclusion principle is not possible." If we apply this to quasibosons, which have integral spin, the theorem simply states that quasibosons cannot obey fermi statistics.

However, one of the basic assumptions of the theorem, space-like commutativity, is also not satisfied by composite integral spin particles since they do not obey Bose commutation relations. Therefore, this theorem does not apply to most of the known integral spin particles (nuclei and molecules with an even number of fermions).

Although the spin-statistics theorem does not apply to composite integral spin particles, Ehrenfest's and Oppenheimer's (Ehrenfest and Oppenheimer, 1931) *approximate* rule does apply. It states that composite particles formed of an even number of fermions obey Bose statistics while those formed of an odd number of fermions obey Fermi statistics. If the spins of the fermions are collinear, the predictions of this rule are the same as those of the spin-statistics theorem. This rule is also supported by the second quantization formalism as an even number of fermion creation operators commute and an odd number anticommute, and the deviations caused by extra terms in the commutation relations are small for tightly bound, well separated particles (Sahlin and Schwartz, 1965).

If integral spin "elementary particles" are formed of multiple fermions, the theorem would also not apply to them. As Messiah and Greenberg (Messiah and Greenberg, 1964) and Von Baeyer (Von Baeyer, 1964) noted long ago, experimental tests are necessary to determine the symmetry of "elementary particles," without recourse to the spin-statistics theorem.

6. COMPARISON WITH EXPERIMENT

Here we look at the quasiboson theory for the photon and the corresponding experimental results. But before that, we will briefly consider He^4 and Cooper pairs, *known* quasibosons.

6.1. Known Quasibosons

Even for the superfluid state of He^4 the molecules are well separated (Huang, 1963) as the interatomic distances are 4×10^{-8} cm while the hard sphere radius is 1×10^{-8} cm. Treating He^4 as a boson is a good approximation as shown by many observables, such as specific heat, ultrasound absorption, and neutron scattering. The Bose–Einstein condensation is also in general agreement with this expectation. However, the Helium ground state potential shows a short-range hard-core repulsion (Feltgen *et al.*, 1982) which is believed to be caused by the Pauli principle.

Since the electron–electron distances for Cooper pairs in a superconductor are about 10^{-8} cm and the pair size is about 10^{-4} cm, one might expect that we could not put many quasibosons in the same state. On the contrary, the Fermi statistics of the components does not prevent us from putting large numbers of fermion pairs into one quasiboson state. As the density of quasibosons increases, the function $F(\mathbf{k})$ spreads in momentum space to allow more quasibosons in the same state. In superconductors, a Bose–Einstein like condensation occurs with a large number of pairs ending up in the lowest energy state. Experimental evidence that the Cooper pairs are not bosons is shown by the energy gap. As Lipkin (1973) noted, "The Pauli principle effect thus produces an energy gap in the excitation spectrum. This effect is characteristic of overlapping fermion pairs, and would be absent if the fermions behaved like simple bosons."

6.2. Photon

The main evidence indicating that photons are bosons comes from the Blackbody radiation experiments which are in agreement with Planck's distribution. We will now calculate the photon distribution for Blackbody radiation by using the second quantization method (Koltun and Eisenberg, 1988), but with a *quasiboson* photon. The atoms in the walls of the cavity are taken to be a two-level system with photons emitted from the upper level β and absorbed at the lower level α. The transition probability for emission of a photon when $n_{\mathbf{p}}$ photons are present is enhanced,

$$w_{\alpha\beta}(n_{\mathbf{p}} + 1 \leftarrow n_{\mathbf{p}}) = (n_{\mathbf{p}} + 1)\left(1 - \frac{n_{\mathbf{p}}}{\Omega}\right) w_{\alpha\beta}(1_{\mathbf{p}} \leftarrow 0), \tag{41}$$

where we have used the first equation of (23). The absorption is also enhanced, but less since we use the second equation of (23)

$$w_{\beta\alpha}(n_{\mathbf{p}} - 1 \leftarrow n_{\mathbf{p}}) = n_{\mathbf{p}}\left(1 - \frac{n_{\mathbf{p}} - 1}{\Omega}\right) w_{\beta\alpha}(0 \leftarrow 1_{\mathbf{p}}). \tag{42}$$

Using the equality,

$$w_{\beta\alpha}(0 \leftarrow 1_{\mathbf{p}}) = w_{\alpha\beta}(1_{\mathbf{p}} \leftarrow 0), \tag{43}$$

of the transition rates (Koltun and Eisenberg, 1988), Eqs. (41) and (42) can be combined to give

$$\frac{w_{\alpha\beta}(n_{\mathbf{p}}+1 \leftarrow n_{\mathbf{p}})}{w_{\beta\alpha}(n_{\mathbf{p}}-1 \leftarrow n_{\mathbf{p}})} = \frac{(n_{\mathbf{p}}+1)\left(1-\frac{n_{\mathbf{p}}}{\Omega}\right)}{n_{\mathbf{p}}\left(1-\frac{(n_{\mathbf{p}}-1)}{\Omega}\right)}. \tag{44}$$

According to Boltzmann's distribution law, the probability of finding the system with energy E is proportional to $e^{-E/kT}$. Thus, the equilibrium between emission and absorption requires that

$$w_{\alpha\beta}(n_{\mathbf{p}}+1 \leftarrow n_{\mathbf{p}})\, e^{-E_\beta/kT} = w_{\beta\alpha}(n_{\mathbf{p}}-1 \leftarrow n_{\mathbf{p}})\, e^{-E_\alpha/kT}, \tag{45}$$

with the photon energy $\omega_p = E_\beta - E_\alpha$. Combining (44) and (45) results in

$$n_{\mathbf{p}} = \frac{2}{u + (u+2)/\Omega + \sqrt{u^2(1+2/\Omega) + (u+2)^2/\Omega^2}}, \tag{46}$$

with $u = e^{\omega_p/kT} - 1$. For $\Omega(\omega_p/kT) \gg 1$, this reduces to

$$n_{\mathbf{p}} = \frac{1}{e^{\omega_p/kT}\left(1+\frac{1}{\Omega}\right) - 1}. \tag{47}$$

For large Ω this approaches Planck's distribution law. The measured quantity in Blackbody radiation experiments is usually W_λ, the spectral emittance as a function of wavelength

$$W_\lambda = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}\left(1+\frac{1}{\Omega}\right) - 1\right)}. \tag{48}$$

The biggest deviations from Planck's law will occur for $\Omega hc/(\lambda kT) < 1$, and in that case Eq. (46) must be used.

We can calculate $\Omega(\mathbf{p}, \mathbf{p})$ by using Eq. (21), but we need a value for the momentum spread k_0. From the uncertainty principle, the momentum spread must be of order $\hbar/\Delta x$ where Δx is the photon wavelength,

$$k_0 \sim \frac{\hbar}{\lambda}. \tag{49}$$

Using (49), Eq. (21) becomes

$$\Omega(\mathbf{p}, \mathbf{p}) = \left(\frac{3}{8\pi}\right)^{3/2} \frac{V}{\lambda^3}. \tag{50}$$

To see the effect of the $1/\Omega(\mathbf{p}, \mathbf{p})$ term, consider the Blackbody radiation experiments of Coblentz (1916) at 1596 K in a cavity of volume 125 cm^3 in the wavelength range 1–6.5 μm. For these conditions, Eq. (47) applies and $1/\Omega(\mathbf{p}, \mathbf{p}) \leq 10^{-9}$, and the maximum deviation from Planck's law is less than one part in 10^{-8}, much too small to be detected. Comparison with other Blackbody radiation experiments also showed that the $1/\Omega(\mathbf{p}, \mathbf{p})$ term is too small to be detected. Unfortunately, we cannot recommend any practical experimental test of Eq. (48).

Another method of determining whether a particle is a quasiboson is by observing the symmetry of two-particle states. As discussed in Section 3, the wave functions of two identical elementary bosons must be symmetric under interchange while the wave functions of two composite quasibosons can be antisymmetric if the two quasibosons are not identical.

According to the theorem of Landau (1948) and Yang (1950), a vector particle (with total angular momentum $= 1$) cannot decay into two photons. This can be seen as follows (Close, 1979): The two photons state must be described in terms of three vectors: the relative momentum $\mathbf{k}$, and the two polarization vectors $\epsilon_{\mathbf{1}}$ and $\epsilon_{\mathbf{2}}$. The state must be bilinear in the polarization vectors. There are just three possibilities:

$$\epsilon_{\mathbf{1}} \times \epsilon_{\mathbf{2}}, \quad (\epsilon_{\mathbf{1}} \cdot \epsilon_{\mathbf{2}})\mathbf{k}, \quad \mathbf{k} \times (\epsilon_{\mathbf{1}} \times \epsilon_{\mathbf{2}}). \tag{51}$$

The last one has zero amplitude because of the transversality condition $\mathbf{k} \cdot \epsilon = 0$. The first two are antisymmetric under an interchange of the two photons. Since two identical bosons or two identical quasibosons are symmetric under interchange, a vector particle cannot decay into two photons, completing the proof.

However, a vector particle can decay into two composite quasiboson photons, if the photons are not identical as in Eq. (31). Many decays of vector particles into two photons are forbidden by charge conjugation invariance. Since n photons transform as $(-1)^n$, two photons will be even under charge conjugation. For example, the ${}^3\mathrm{S}_1$, and ${}^1\mathrm{P}_1$ states of positronium have $C = -1$ and thus cannot decay into two photons. The ${}^3\mathrm{P}_1$ state of positronium has $C = +1$ and can decay into two nonidentical photons, providing a test of this theory.

Most of the vector mesons cannot decay into two photons because of charge conjugation invariance, but some axial vector mesons such as $f_1(1285)$, $f_1(1420)$, and $\chi_{c1}(1P)$ with $J^{\mathrm{PC}} = 1^{++}$ can. Detection of such decays can provide evidence that the photon is a composite particle with nonidentical forms.

7. CONCLUSIONS

It is our conjecture that all integral spin particles are quasibosons, composed of fermions. This is based on the observation that most known integral-spin particles are quasibosons which behave so similar to bosons that it is difficult to detect the non-Bose effects caused by the underlying fermions. The experimental results regarding the photon, which is usually held up as the exemplar boson, are inconclusive. It was shown in Section 6.2 that the Blackbody radiation from quasiboson photons is so similar to Planck's distribution that the difference could not have been detected in any existing experiment. It was shown in Sections 4 and 5 that the spin-statistics theorem does not apply to composite particles because their fields are nonlocal.

In this paper, we have only considered the photon for it is usually considered to be the model boson. Tests to determine whether two photons are always symmetric under interchange were discussed in Section 6.2.

Although the commutation relations, Eqs. (3) and (9), are more complex than those for bosons, it is really a simpler picture of matter in that there exists only fermions and composite particles formed of fermions. The old approximation is still valid: If a composite particle is formed of an odd number of fermions, use Fermi statistics, and if a composite particle is formed of an even number of fermions, use Bose statistics. Of course, in some cases, such as Cooper pairs, this is not a good approximation.

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